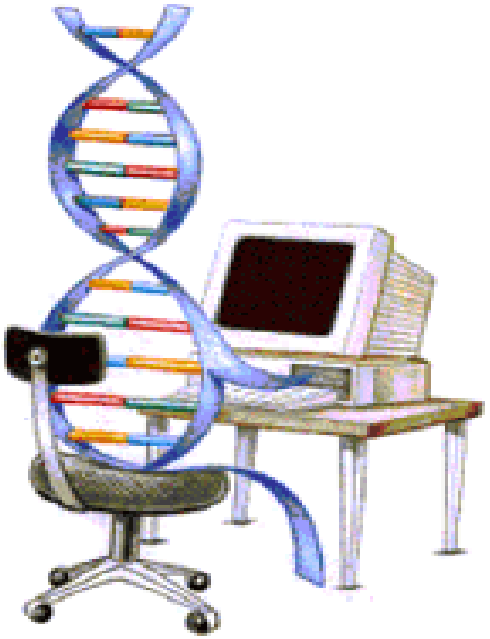
Genetic Algorithms

Lecture 22



Overview

- Introduction To Genetic Algorithms (GAs)
- GA Operators and Parameters
- GA Hello world
- GA for traveling salesman problem
- GA for best rule finding

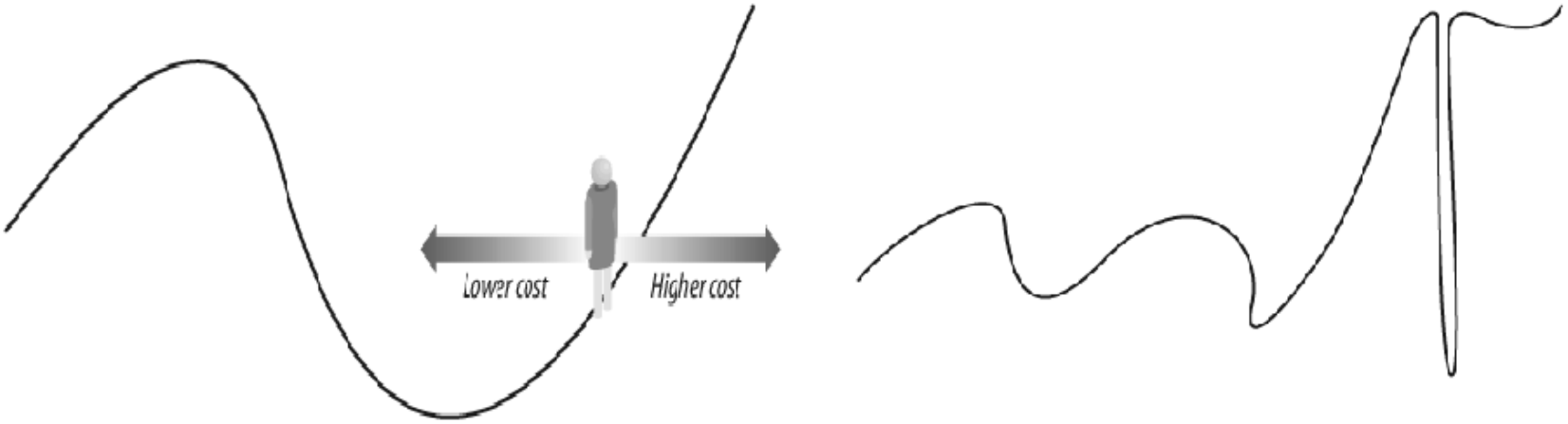
Introduction To Genetic Algorithms (GAs)

History Of Genetic Algorithms

- "Evolutionary Computing" was introduced in the 1960s by **I. Rechenberg**.
- John Holland wrote the first book on Genetic Algorithms '*Adaptation in Natural and Artificial Systems*' in 1975.
- In 1992 **John Koza** used genetic algorithm to evolve programs to perform certain tasks. He called his method "Genetic Programming".

When to use genetic algorithms
John Holland (1975)

- Optimization: minimize (maximize) some function f(x) over all possible values of variables x in X
- A brute force: examining every possible combination of x in X in order to determine the element for which f is optimal: infeasible
- Optimization techniques are heuristic.
- The problem of local maximum (minimum).



Mutation introduces randomness in the method to get out of trap

Sample Problem

The **Traveling Salesman Problem** is defined as:

'We are given a set of cities and a symmetric distance matrix that indicates the cost of travel from each city to every other city.

*The goal is to find **the shortest circular tour,** visiting every city exactly once, so as to **minimize the total travel cost**, which includes the cost of traveling from the last city back to the first city'.*

What Are Genetic Algorithms (GAs)?

Genetic Algorithms are *search* and *optimization* techniques based on Darwin's Principle of *Natural Selection*.

Why GAs

- Evolution is known to be a successful, robust method to produce adaptations (solutions) to different environments (problems)
- GAs can search a very large space of hypotheses containing complex interacting parts
- GAs are inherently parallelizable

Darwin's Principle Of Natural Selection

IF there are organisms that reproduce, and

IF offsprings inherit traits from their progenitors, and

IF there is variability of traits, and

IF the environment cannot support all members of a growing population,

THEN those members of the population with less-adaptive traits (determined by the environment) will die out, and

THEN those members with more-adaptive traits (determined by the environment) will thrive

The result is the evolution of species.

Basic Idea Of Principle Of Natural Selection

"Select The Best, Discard The Rest"

An Example of Natural Selection

- **Giraffes have long necks**.

Giraffes with slightly longer necks could feed on leaves of higher branches when all lower ones had been eaten off.

→ They had a better chance of survival.

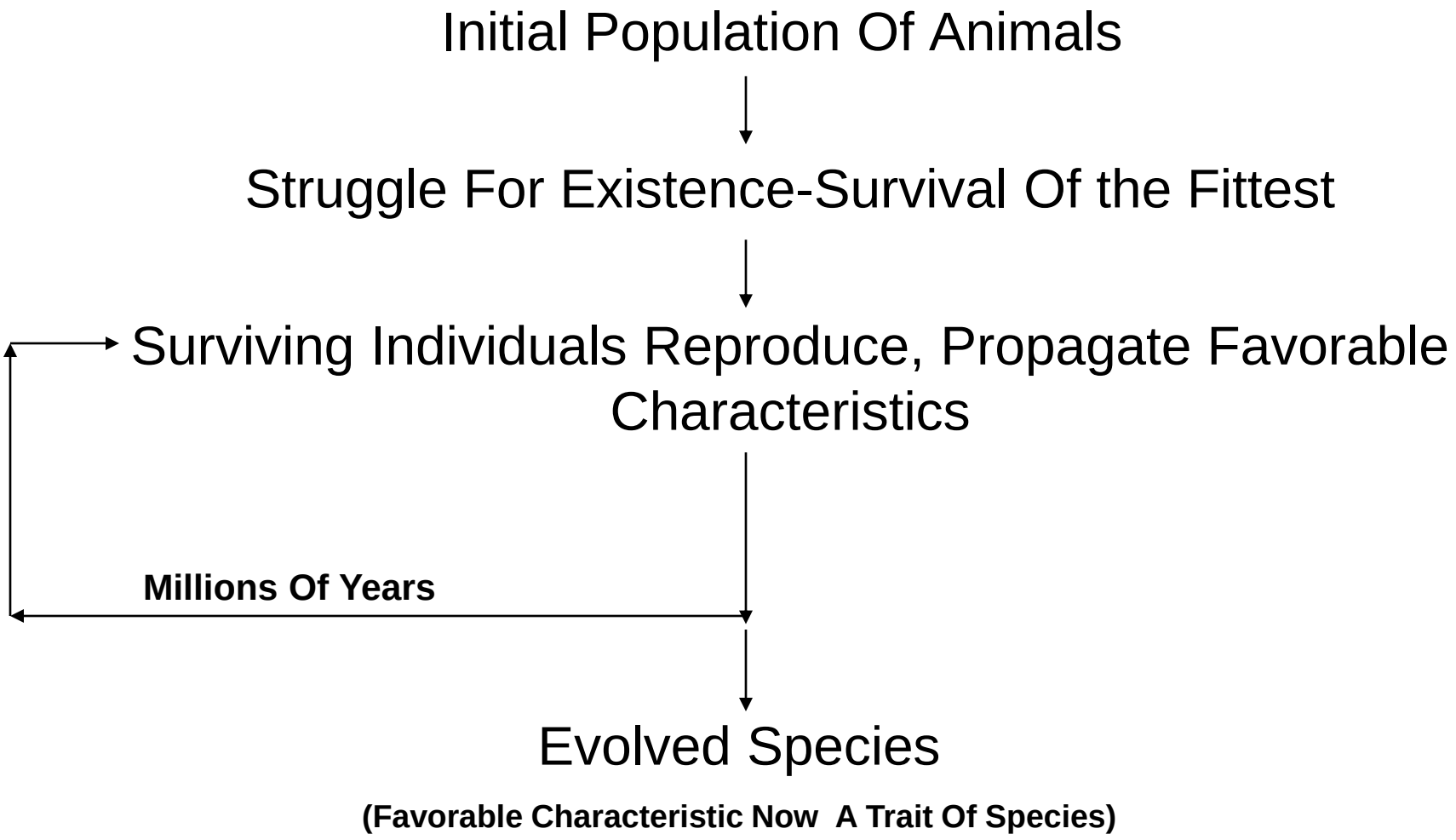
→ Favorable characteristic propagated through generations of giraffes.

→ Now, evolved species has long necks.

NOTE: Longer necks may have been a deviant characteristic (mutation) initially but since it was favorable, was propagated over generations. Now an established trait.

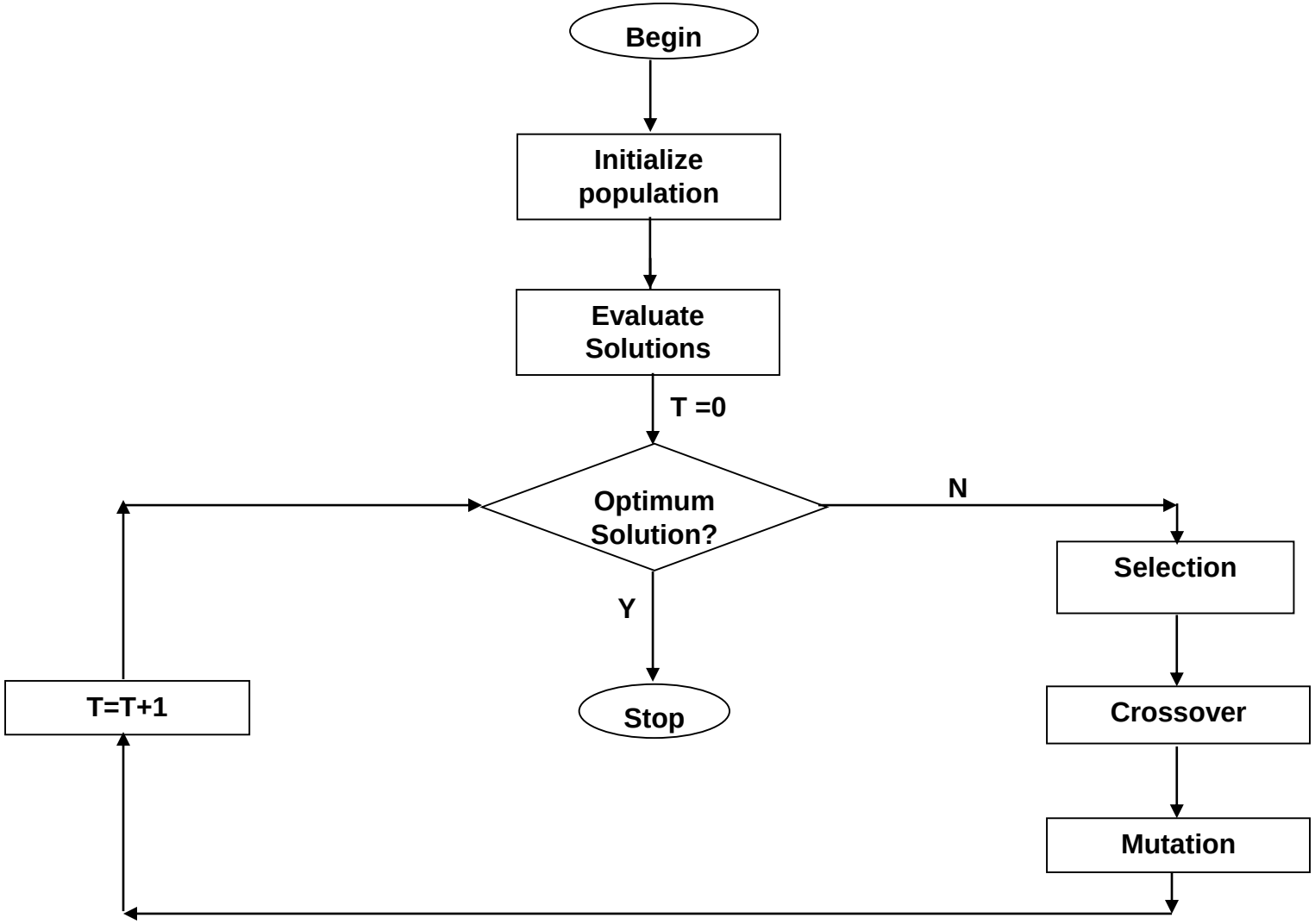
So, some mutations are beneficial.

Evolution Through Natural Selection



Genetic Algorithms Implement Optimization Strategies By Simulating Evolution Of Species Through Natural Selection.

Working Mechanism Of GAs



Simple Genetic Algorithm

```
Simple_Genetic_Algorithm()
   {
        Initialize the Population;
        Calculate Fitness Function;

        While(Fitness Value != Optimal Value)
        {
              Selection;
              //Natural Selection, Survival Of Fittest

              Crossover;
              //Reproduction

              Mutation;
              //Mutation
              Calculate Fitness Function;
        }
   }
```

Nature to Computer Mapping

Nature	Computer
Population	Set of initial hypotheses (possible solutions).
Individual	Solution to a problem.
Fitness	Quality of a solution.
Chromosome	Encoding for a Solution.
Gene	Part of the encoding of a solution.
Reproduction (crossover)	Crossover

GA Operators and Parameters

1. *Encoding*
2. *Fitness function*
3. *Selection*
4. *Mating (crossover)*
5. *Mutation*

1. Encoding

*The process of representing the solution in the form of a **string** that conveys the necessary information.*

- Just as in a chromosome, each gene controls a particular characteristic of the individual, similarly, each bit in the string represents a characteristic of the solution.

Encoding Methods

- **Binary Encoding –** Most common method of encoding. Chromosomes are strings of 1s and 0s and each position in the chromosome represents a particular characteristic of the problem.

Chromosome A	10110010110011100101
Chromosome B	11111110000000011111

Representing Hypotheses

Represent

(Outlook = Overcast $\vee$ Rain) $\wedge$ (Wind = Strong)

by

Outlook	Wind
011	**10**

Represent

IF Wind = Strong THEN PlayTennis = yes

by

Outlook	Wind	PlayTennis
111	**10**	**10**

Don't care for Outlook here.

Encoding Methods (contd.)

- **Permutation Encoding –** Useful in scheduling problems such as the Traveling Salesman Problem (TSP). Example. In TSP, every chromosome is a string of numbers, each of which represents a city to be visited in this order.

Chromosome A	1 5 3 2 6 4 7 9 8
Chromosome B	8 5 6 7 2 3 1 4 9

Encoding Methods (contd.)

- **Value Encoding –** Used in problems where complicated values, such as real numbers, are used and where binary encoding would not suffice.

  Good for some problems, but *often necessary to develop some specific crossover and mutation techniques for these chromosomes.*

Chromosome A	1.235 5.323 0.454 2.321 2.454
Chromosome B	(left), (back), (left), (right), (forward)

2. Fitness Function

A fitness function quantifies the optimality of a solution (chromosome) so that that particular solution may be ranked against all the other solutions.

- A fitness value is assigned to each solution depending on how close it actually is to solving the problem.
- Ideal fitness function correlates closely to goal + quickly computable.
- Example. In TSP, f(x) is sum of distances between the cities in solution. The lesser the value, the fitter the solution is.

3. Selection

The process that determines which solutions are to be preserved and allowed to reproduce and which ones deserve to die out.

- The primary objective of the recombination operator is to emphasize the good solutions and eliminate the bad solutions in a population, while keeping the population size constant.
- “Selects The Best, Discards The Rest”.

Elite Selection

- Sort solutions by fitness (descending).
- Make multiple copies of the top solutions (parthenogenesis – cloning).
- Eliminate bad solutions from the population so that multiple copies of good solutions can be placed in the population.

Roulette Wheel Selection

- Each current string in the population has a slot assigned to it which is in proportion to it's fitness.

- We spin the weighted *roulette wheel* thus defined *n* times (where *n* is the total number of solutions).

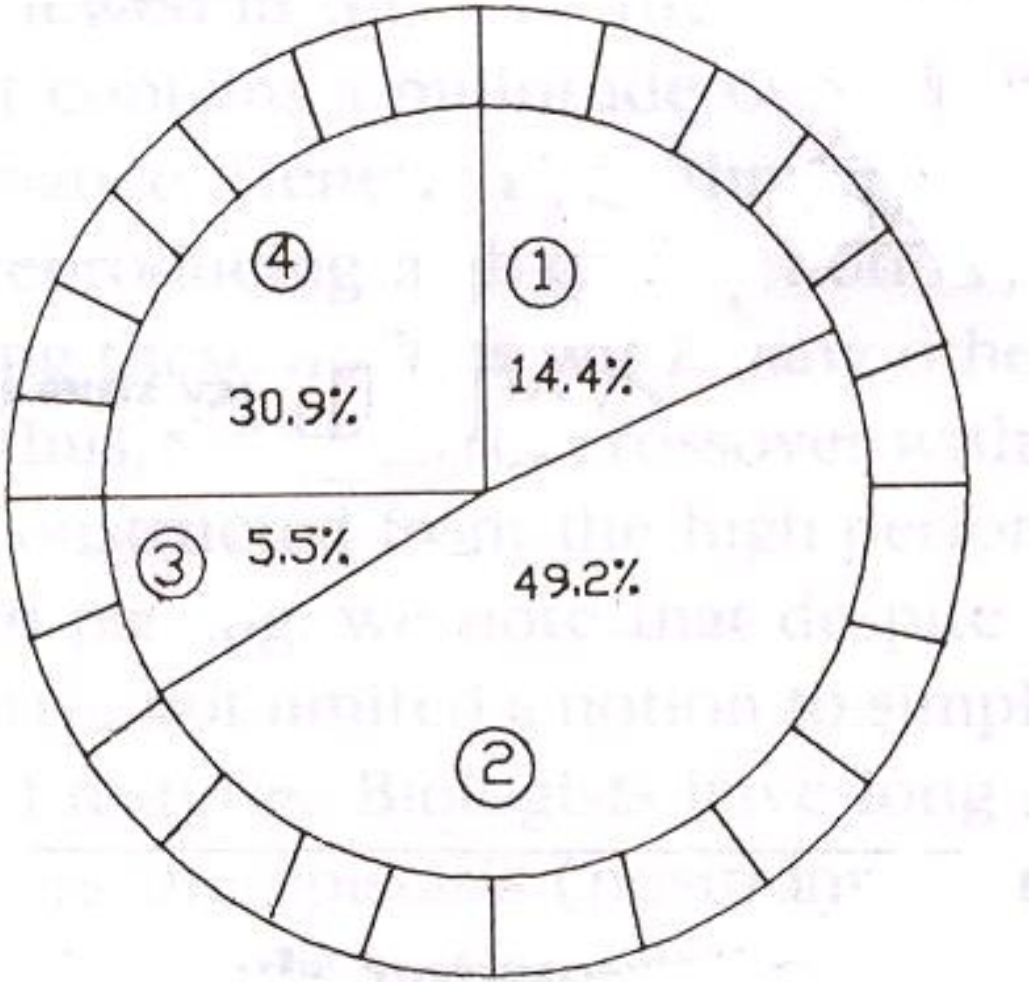
- Each time Roulette Wheel stops, the string corresponding to that slot is created.

Strings that are fitter are assigned a larger slot and hence have a better chance of appearing in the new population.

Example Of Roulette Wheel Selection

No.	String	Fitness	% Of Total
1	01101	169	14.4
2	11000	576	49.2
3	01000	64	5.5
4	10011	361	30.9
Total		1170	100.0

Roulette Wheel For Example



Tournament selection

- **Tournament Selection**: Two members are chosen at random from a population.
  - With some predefined probability p the more fit of these two is then selected, and with probability 1-p the less fit hypothesis is selected.

Sometimes TS yields a more diverse population that RS.

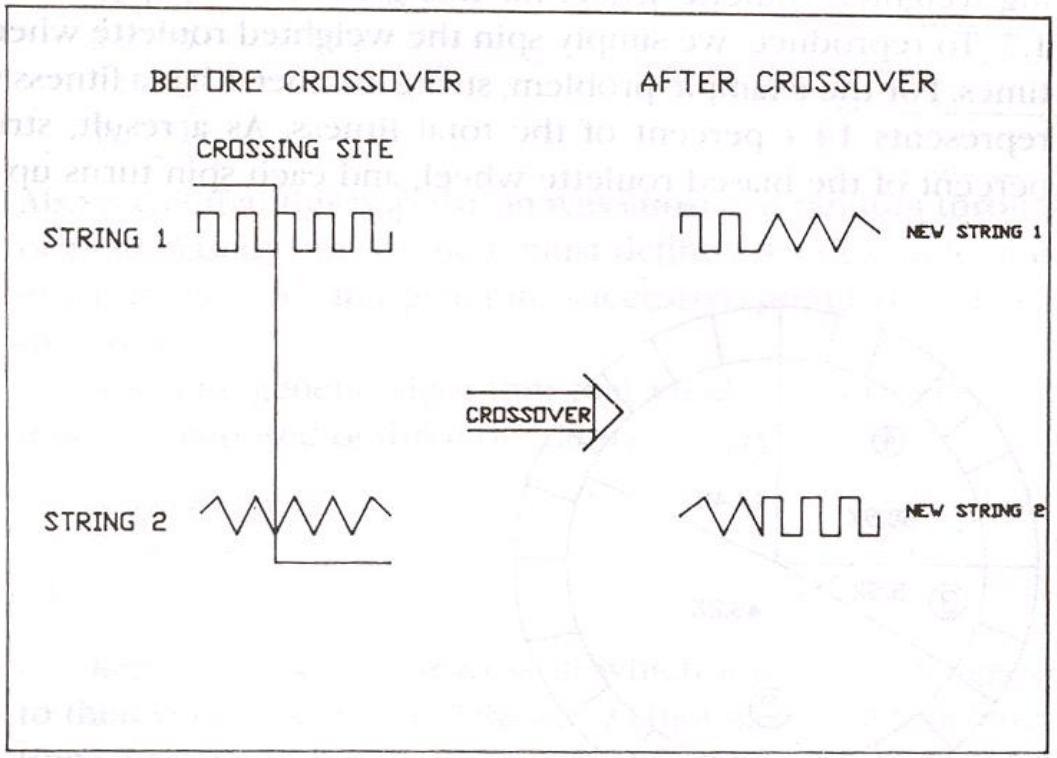
4. Mating (Crossover)

It is the process in which two chromosomes (strings) combine their genetic material (bits) to produce a new offspring which possesses both their characteristics.

- *Two strings are picked from the mating pool (or from elite) at random to cross over.*
- *The method chosen depends on the Encoding Method.*

Crossover Methods

- **Single Point Crossover-** A random point is chosen on the individual chromosomes (strings) and the genetic material is exchanged at this point.



Crossover Methods (contd.)

- **Single Point Crossover**

Chromosome1	11011 | 00100110110
Chromosome 2	11011 | 11000011110
Offspring 1	11011 | 11000011110
Offspring 2	11011 | 00100110110

Crossover Methods (contd.)

- **Two-Point Crossover-** Two random points are chosen on the individual chromosomes (strings) and the genetic material is exchanged at these points.

Chromosome1	**11011 | 00100 | 110110**
Chromosome 2	**10101 | 11000 | 011110**
Offspring 1	**10101 | 00100 | 011110**
Offspring 2	**11011 | 11000 | 110110**

NOTE: These chromosomes are different from the last example.

Crossover Methods (contd.)

- **Uniform Crossover-** Each gene (bit) is selected randomly from one of the corresponding genes of the parent chromosomes.

Chromosome1	**11011 | 00100 | 110110**
Chromosome 2	**10101 | 11000 | 011110**
Offspring	**10111 | 00000 | 110110**

NOTE: Uniform Crossover yields ONLY 1 offspring.

Crossover (contd.)

- Crossover between 2 good solutions **MAY NOT ALWAYS** yield a better or as good a solution.
- Since parents are good, probability of the child being good is high.
- If offspring is not good (poor solution), it will be removed in the next iteration during “Selection”.

Elitism

Elitism is a method which copies the best chromosome to the new offspring population before crossover and mutation.

- When creating a new population by crossover or mutation the best chromosome might be lost.
- Forces GAs to retain some number of the best individuals at each generation.
- Has been found that elitism significantly improves performance.

5. Mutation

It is the process by which a string is deliberately changed so as to maintain diversity in the population set.

We saw in the giraffes' example, that mutations could be beneficial.

Mutation Probability- determines how often the parts of a chromosome will be mutated.

Example Of Mutation

- For chromosomes using Binary Encoding, randomly selected bits are inverted.

Offspring	**11011 00100 10110**
Mutated Offspring	**11010 00100 10110**

NOTE: The number of bits to be inverted depends on the Mutation Probability.

Advantages Of GAs

- **Global Search Methods:** GAs search for the function optimum starting from a *population of points* of the function domain, not a single one. This characteristic suggests that GAs are global search methods. They can, in fact, climb many peaks in parallel, reducing the probability of finding local minima, which is one of the drawbacks of traditional optimization methods.

Advantages of GAs (contd.)

- **Blind Search Methods:** GAs only use the information about the *objective function*. They do not require knowledge of the first derivative or any other auxiliary information, allowing a number of problems to be solved without the need to formulate restrictive assumptions. For this reason, GAs are often called blind search methods.

Advantages of GAs (contd.)

- **GAs use probabilistic transition rules** during iterations, unlike the traditional methods that use fixed transition rules.

  This makes them more robust and applicable to a large range of problems.

Advantages of GAs (contd.)

- **GAs can be easily used in *parallel machines*-** Since in real-world design optimization problems, most computational time is spent in evaluating a solution, with multiple processors all solutions in a population can be evaluated in a distributed manner. This reduces the overall computational time substantially.

GA Hello World

- Simple example: random population of strings evolves into a predefined template “Hello World”
- For simplicity:
  - random strings have the same length as the target string
  - Fitness function is calculated as the closeness of the given string to the target string

Encoding possible solutions

- Sequences of characters
- Start from a population of random strings

```
//build initial population
ArrayList <String> population=new ArrayList<String>();
Random generator = new Random();

for(int i=0;i<population_size;i++)
{
        char [] newIndividual=new char[target_length];
        for(int j=0;j<target_length;j++)
        newIndividual[j]=(char)(generator.nextInt(32000)%90 + 32);
        population.add(new String(newIndividual));
}

return population;
```

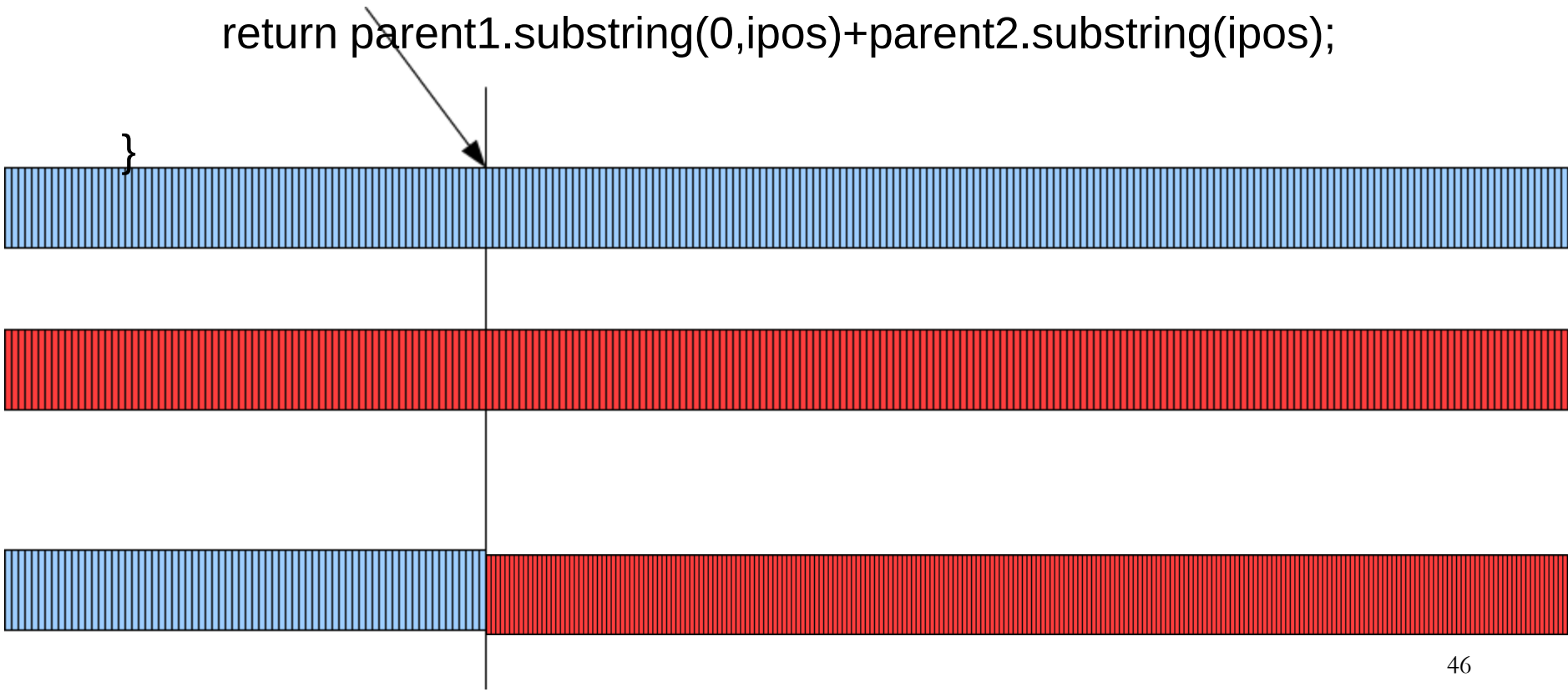
Fitness function

- Compares each member of the population with the target string. It adds up the differences between the characters and uses the cumulative sum as the fitness value (therefore, the lower the value, the better).

```
public  double string_fitness (String individual)
{
        double fitness=0;
        for (int ipos=0; ipos<target_length;ipos++)
           fitness+=Math.abs( individual.charAt(ipos) -
                    TARGET_STRING.charAt(ipos)) ;
        return fitness;
}
```

Mate operation (crossover)

```
public String string_crossover(String parent1, String parent2)
    {
        Random generator = new Random();
        int ipos = generator.nextInt( target_length-1 );
        return parent1.substring(0,ipos)+parent2.substring(ipos);
    }
```



Mutation Operation

```
public  String mutate_string(String individual)
    {
        Random generator = new Random();
        int ipos = generator.nextInt( target_length );

        //mutation changes character at random to any available
    character from 32 (space) to 90 (Z)

        char rchar=(char)(generator.nextInt(32000)%90 + 32);

        char [] indToChars=individual.toCharArray();
        indToChars[ipos]=rchar;

        String mutatedIndividual=new String(indToChars);

        return mutatedIndividual;
    }
```

Selection: Elitism

```
currentIteration++;
individual_scores=new ArrayList<Individual>();
for(int j=0;j<population.size();j++)
{
        String currString=population.get(j);

        Individual curr=new Individual();
        curr.str=currString;
        curr.fitness=string_fitness(currString);

        individual_scores.add(curr);
}
Collections.sort(individual_scores);
```

Selection: Elitism (Cont.)

```
// Start with the pure winners
for(int k=0;k<top_elite;k++)
        population.add(individual_scores.get(k).str);

// Add mutated and bred forms of the winners
Random generator = new Random();
while (population.size()<original_population_size)
{
        //mutate with the mutation_probability
        double r = generator.nextDouble();
        if (r<mutation_probability) {
                String mutated=mutate_string(population.get(c));
                population.add(mutated);
        }
                else
```

Selection: Elitism (Cont.)

```
// Start with the pure winners
for(int k=0;k<top_elite;k++)
        population.add(individual_scores.get(k).str);

// Add mutated and bred forms of the winners
Random generator = new Random();
while (population.size()<original_population_size)
{ …
        else      {
                  //Crossover
                  int c1=generator.nextInt(top_elite);
                  int c2=generator.nextInt(top_elite);
        String child= string_crossover(population.get(c1),population.get(c2));
                           population.add(child);
        }
}
```

Random optimizer: for comparison

- Random searching isn't a very good optimization method, but it makes it easy to understand exactly what all the algorithms are trying to do, and it also serves as a baseline so you can see if the other algorithms are doing a good job.
- The random optimizer randomly generates 202,800 random guesses and applies a fitness function for each guess. It keeps track of the best guess (the one with the lowest cost) and returns it.

Genetic Algorithms To Solve The Traveling Salesman Problem (TSP)

The Problem

The **Traveling Salesman Problem** is defined as:

'We are given a set of cities and a symmetric distance matrix that indicates the cost of travel from each city to every other city.

*The goal is to find **the shortest circular tour,** visiting every city exactly once, so as to **minimize the total travel cost**, which includes the cost of traveling from the last city back to the first city'.*

Encoding

- Encode every city with an integer .

- Consider 6 Indian cities –

  Mumbai, Nagpur , Calcutta, Delhi , Bangalore and Chennai and assign a number to each.

Mumbai → 1
Nagpur → 2
Calcutta → 3
Delhi → 4
Bangalore → 5
Chennai → 6

Encoding (contd.)

- Thus a path would be represented as a sequence of integers from 1 to 6.
- The path [1 2 3 4 5 6] represents a path from Mumbai to Nagpur, Nagpur to Calcutta, Calcutta to Delhi, Delhi to Bangalore, Bangalore to Chennai, and finally from Chennai to Mumbai.
- This is an example of **Permutation Encoding** as the position of the elements determines the fitness of the solution.

Fitness Function

- The fitness function will be the total cost of the tour represented by each chromosome.
- This can be calculated as the sum of the distances traversed in each travel segment.

The lesser the sum, the fitter the solution represented by that chromosome.

Distance/Cost Matrix For TSP

	1	2	3	4	5	6
1	0	863	1987	1407	998	1369
2	863	0	1124	1012	1049	1083
3	1987	1124	0	1461	1881	1676
4	1407	1012	1461	0	2061	2095
5	998	1049	1881	2061	0	331
6	1369	1083	1676	2095	331	0

Cost matrix for six city example.

Distances in Kilometers

Fitness Function (contd.)

- So, for a chromosome [4 1 3 2 5 6], the total cost of travel or fitness will be calculated as shown below

- Fitness = *1407 + 1987 + 1124 + 1049 + 331+ 2095*
  = *7993 kms.*

- Since our objective is to Minimize the distance, the lesser the total distance, the fitter the solution.

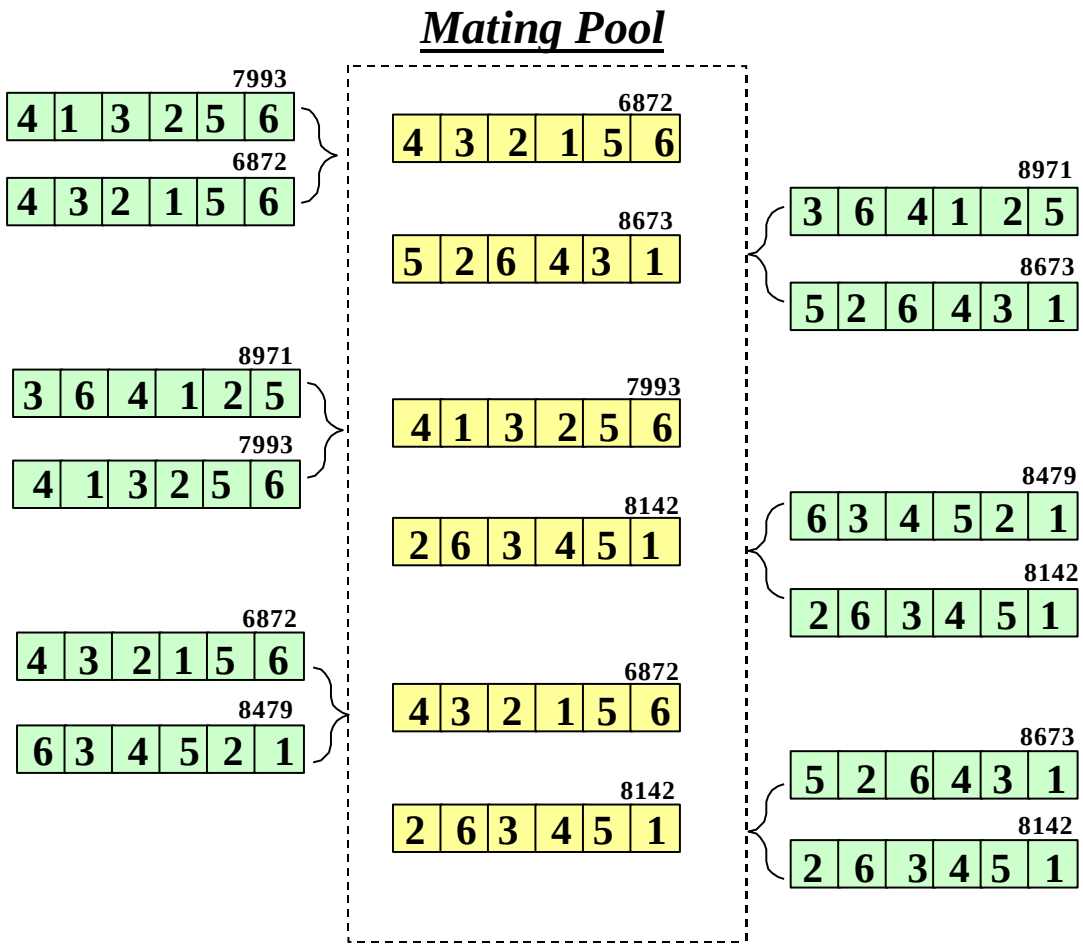
Selection Operator

Use ***Tournament Selection***.

As the name suggests *tournaments* are played between two solutions and the better solution is chosen and placed in the *mating pool*.

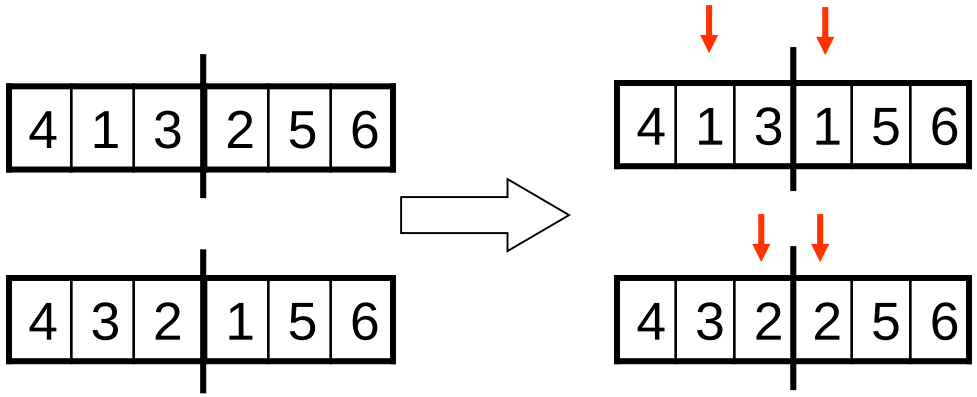
Two other solutions are picked again and another slot in the *mating pool* is filled up with the better solution.

Tournament Selection (contd.)



Why we cannot use single-point crossover:

- Single point crossover method randomly selects a crossover point in the string and swaps the substrings.
- This may produce some invalid offsprings as shown below.



Crossover Operator

- Use the *Enhanced Edge Recombination* operator (T.Starkweather, et al, *'A Comparison of Genetic Sequencing Operators,* International Conference of GAs, 1991) .
- This operator is different from other genetic sequencing operators in that it emphasizes *adjacency information* instead of the order or position of items in the sequence.
- The algorithm for the Edge-Recombination Operator involves constructing an Edge Table first.

Edge Table

The *Edge Table* is an *adjacency table* that lists links *into* and *out of* a city found in the two parent sequences.

If an item is already in the edge table and we are trying to insert it again, that element of a sequence must be a *common edge* and is represented by inverting it's sign.

Finding The Edge Table

Parent 1	4	1	3	2	5	6
Parent 2	4	3	2	1	5	6

1	4	3	2	5
2	-3	5	1	
3	1	-2	4	
4	-6	1	3	
5	1	2	-6	
6	-5	-4		

Enhanced Edge Recombination Algorithm

1. Choose the initial city from one of the two parent tours. (It can be chosen randomly as according to criteria outlined in *step 4*). This is the *current city.*

2. Remove all occurrences of the *current city* from the left hand side of the edge table.(These can be found by referring to the edge-list for the *current city*).

3. If the *current city* has entries in it's edge-list, go to s*tep 4* otherwise go to *step 5*.

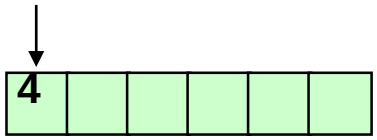
4. Determine which of the cities in the edge-list of the *current city* has the fewest entries in it's own edge-list. The city with fewest entries becomes the *current city*. In case a negative integer is present, it is given preference. Ties are broken randomly. Go to *step 2.*

5. If there are no remaining *unvisited* cities, then *stop.* Otherwise, randomly choose an *unvisited* city and go to *step 2.*

Example Of Enhanced Edge Recombination Operator

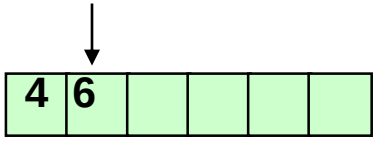
Step 1

1	4	3	2	5
2	-3	5	1	
3	1	-2	4	
4	-6	1	3	
5	3	2	-6	
6	-5	-4		



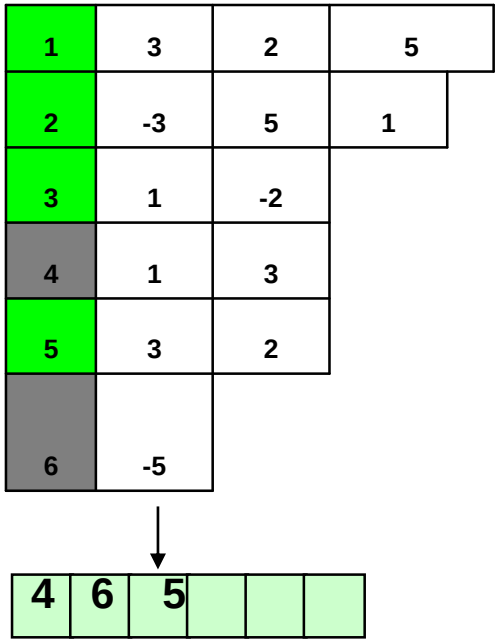
Step 2

1	3	2	5
2	-3	5	1
3	1	-2	
4	-6	1	3
5	3	2	-6
6	-5		

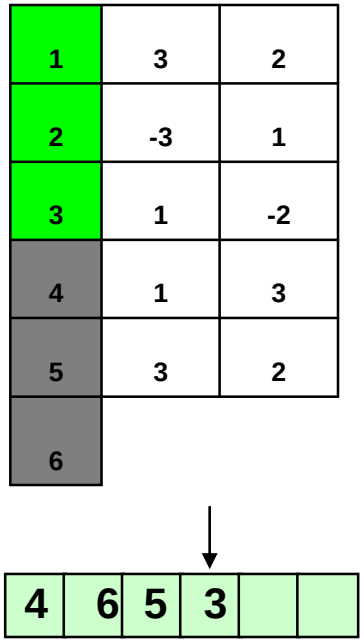


Example Of Enhanced Edge Recombination Operator (contd.)

Step 3

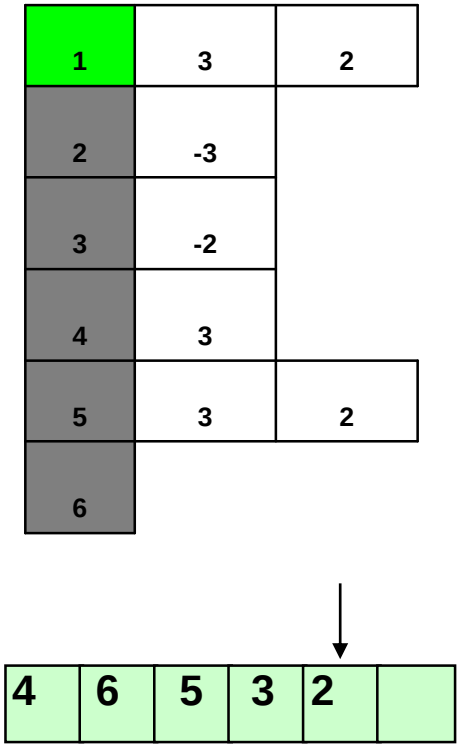


Step 4

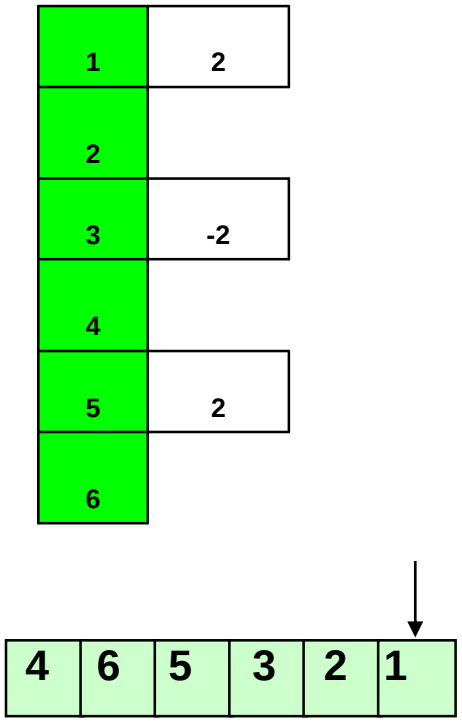


Example Of Enhanced Edge Recombination Operator (contd.)

Step 5

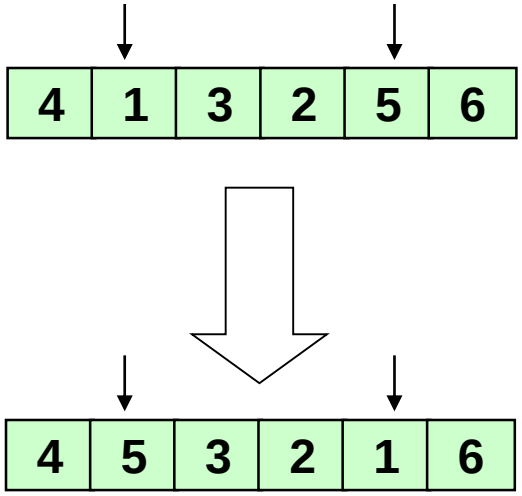


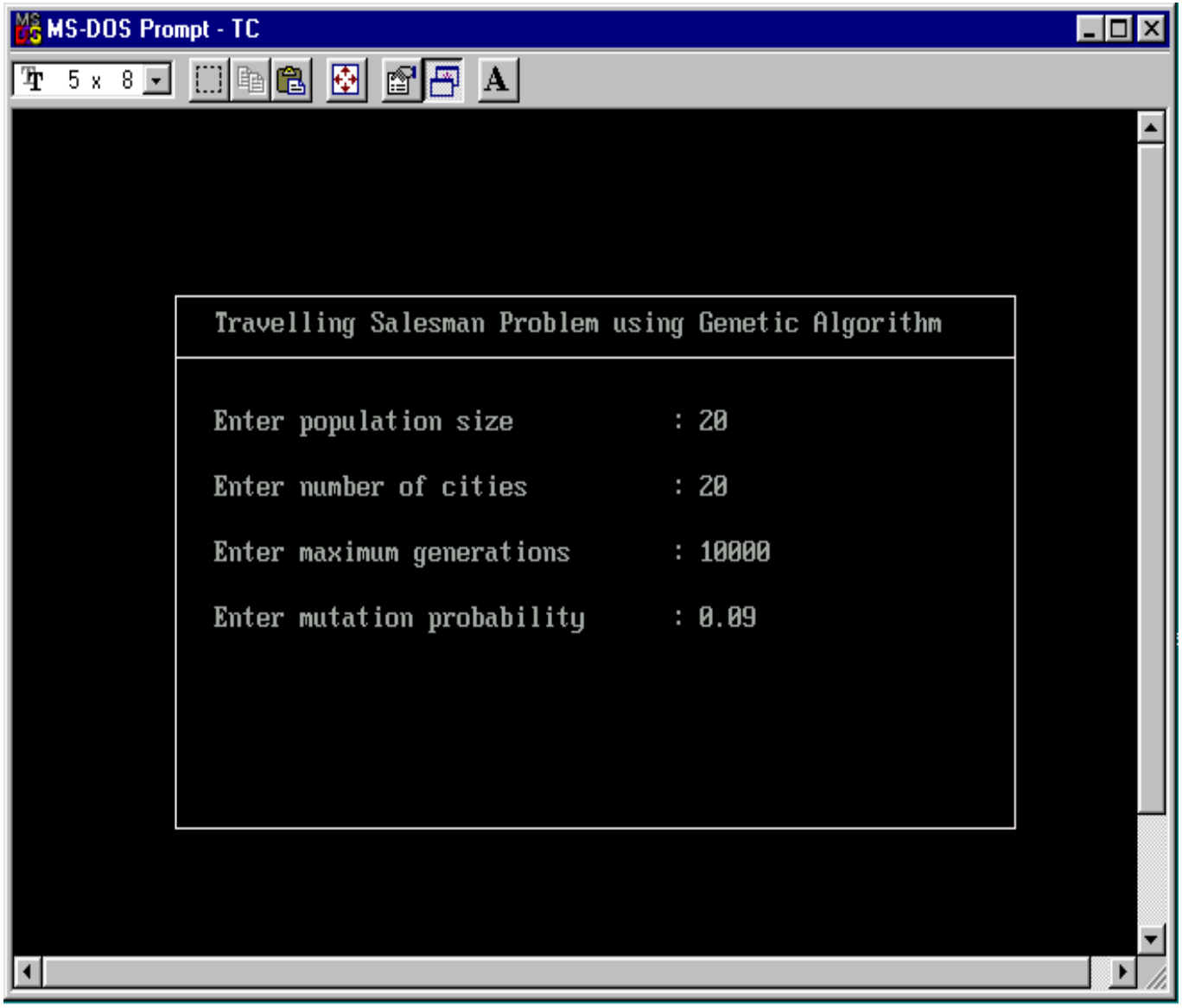
Step 6



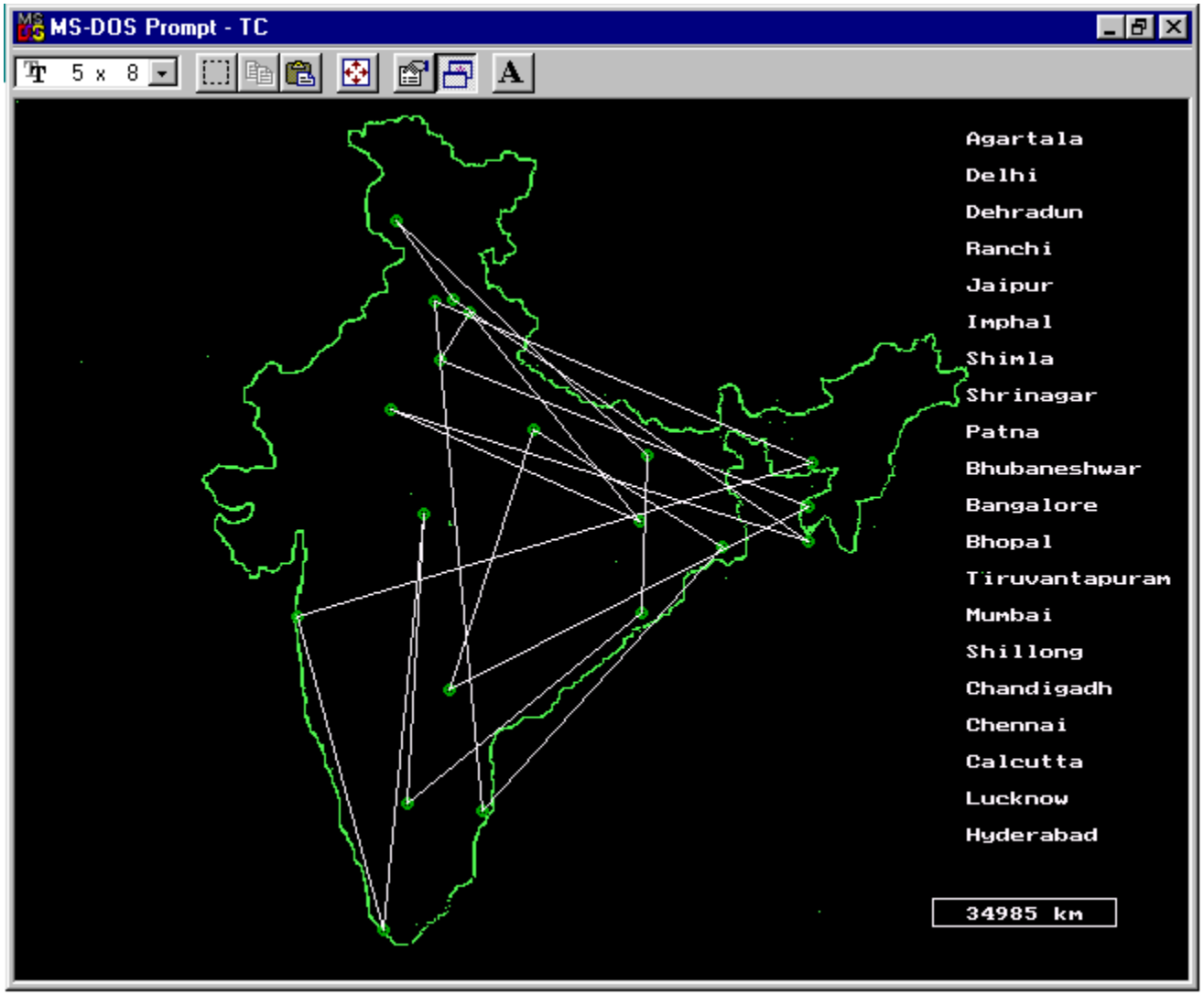
Mutation Operator

- The mutation operator induces a change in the solution, so as to maintain diversity in the population and prevent *Premature Convergence*.
- We mutate the string by randomly selecting any two cities and interchanging their positions in the solution, thus giving rise to a new tour.

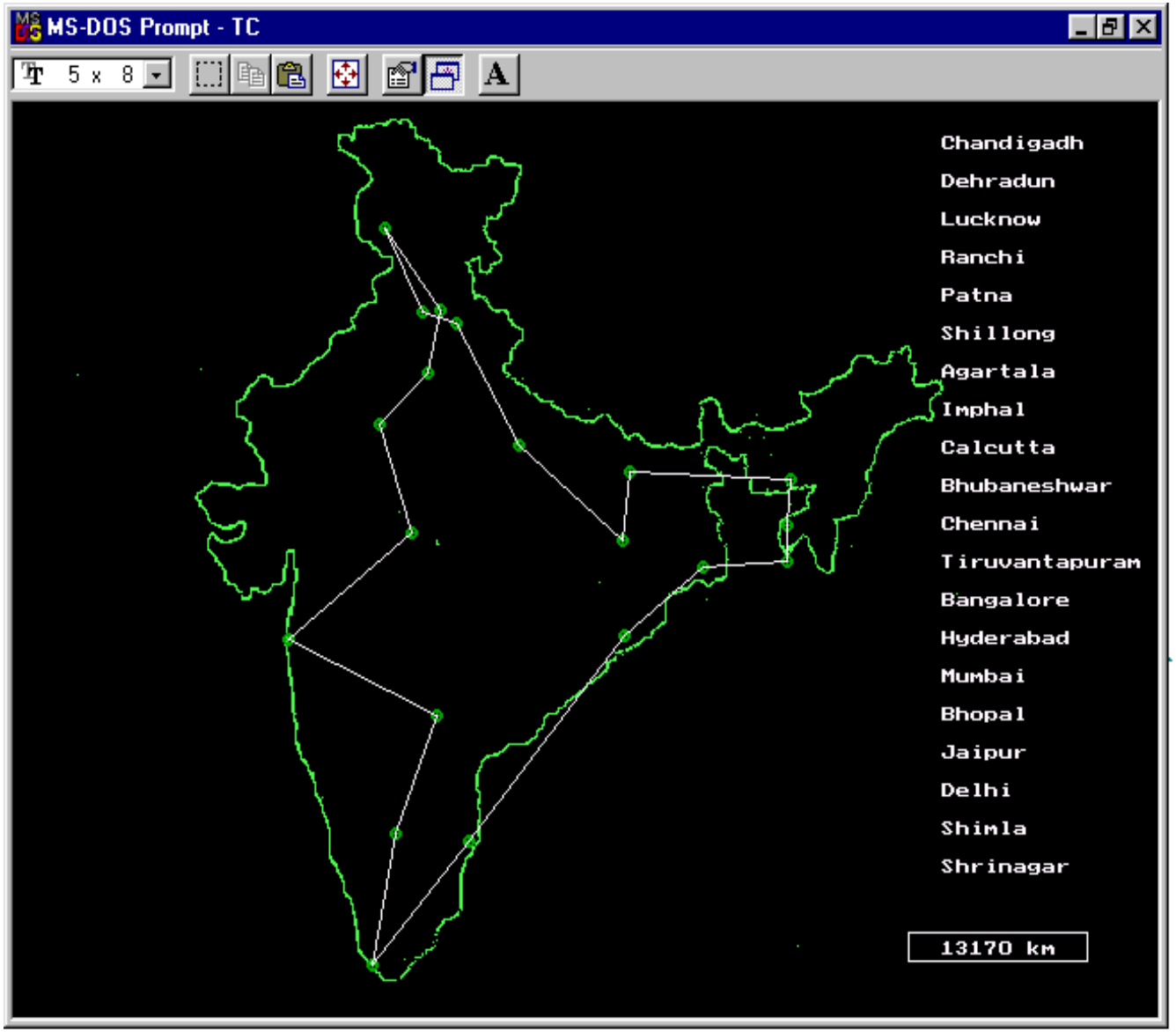




Input to the Program



Initial Output For 20 cities : Distance=34985 km
Initial Population



Final Output For 20 cities : Distance=13170 km
Generation 4786

Genetic Algorithms To Find Best Rules

Representing Hypotheses (Binary encoding)

Represent

(Outlook = Overcast $\vee$ Rain) $\wedge$ (Wind = Strong)

by

Outlook	Wind
011	**10**

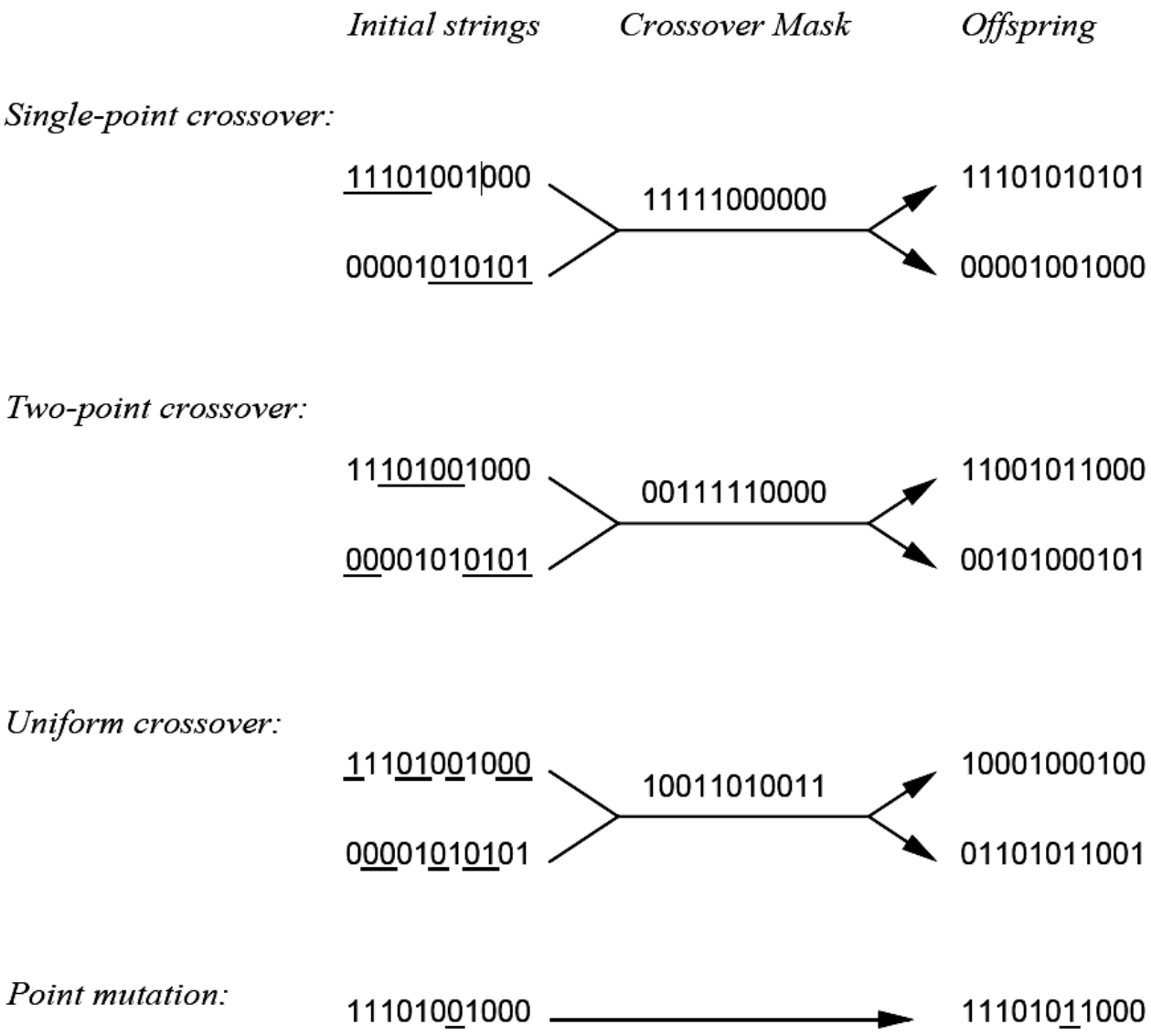
Represent

IF Wind = Strong THEN PlayTennis = yes

by

Outlook	Wind	PlayTennis
111	**10**	**10**

Crossover and mutation



GABIL [DeJong et al. 1993]

- Learn a set of rules

Representation:

- Each hypothesis is a set of rules
- To represent a set of rules, the bit-string representation of individual rules are concatenated

Example

IF a_1 = T AND a_2 = F THEN c = T;

IF a_2 = T THEN c = F

a_1	a_2	c	a_1	a_2	c
10	01	1	11	10	0

Fitness function for a set of rules

Fitness function:

Fitness(h) = $(\text{correct}(h))^2$

correct(h): the percent of all training examples correctly classified

Mutation and crossover

- Use the standard mutation operator
- Crossover: extension of the two-point crossover operator
  - want variable length rule sets
  - want only well-formed bitstring hypotheses

- Crossover with variable-length bitstrings

  1. choose two crossover points for h1. Let d1 (d2) be the distance to the rule boundary immediately to its left.

  2. now restrict points in h2 to those that have the same d1 and d2 value

Crossover example

	a_1	a_2	c	a_1	a_2	c
h1 :	10	01	1	11	10	0
h2 :	01	11	0	10	01	0

Suppose crossover points for h1, are after bits 1, 8 (d1=1;d2=3)

	a_1	a_2	c	a_1	a_2	c
h1 :	1[0	01	1	11	1]0	0

Allowed pairs of crossover points for h2 are <1;3>, <1;8>, <6;8>.

If pair <1;3> is chosen,

	a_1	a_2	c	a_1	a_2	c
h2 :	0[1	1]1	0	10	01	0

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the result is:

Crossover example (cont.)

	a_1	a_2	c	a_1	a_2	c
h1 :	**1**[0	01	1	11	1]**0**	**0**
h2 :	0[**1**	**1**]1	0	10	01	0

	a_1	a_2	c
c1 :	**11**	**10**	**0**

	a_1	a_2	c	a_1	a_2	c	a_1	a_2	c
c2 :	00	01	1	11	11	0	10	01	0

Summary

- *Genetic Algorithms* (GAs) implement optimization strategies based on simulation of the natural law of evolution of a species by *natural selection*

- The basic GA Operators are:
  - Encoding
  - Selection
  - Crossover
  - Mutation

- GAs have been applied to a variety of function optimization problems, and have been shown to be highly effective in searching a large, poorly defined search space even in the presence of difficulties such as high-dimensionality, multi-modality, discontinuity and noise.